One lexeme, many classes: inflection class systems as lattices

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Introduction

In some inflectional systems, the same morphosyntactic properties can be expressed differently across lexemes. Descriptions of the resulting inflection classes (declensions or conjugations) can take several forms. The simplest possibility is to use a partition of the set of lexemes into classes, as in Figure 1a. Possible partitions will differ in their granularities. Pedagogical grammars are often content with giving a broad classification in major classes. At the other end of the spectrum, various studies (e.g. Stump & Finkel 2013) presuppose a classification into numerous fine-grained classes.

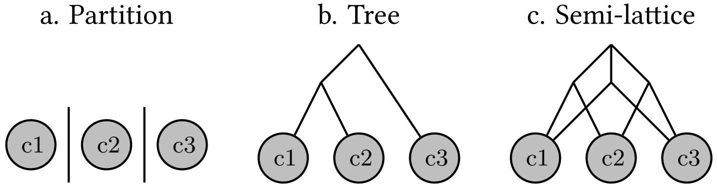

Figure 1: Three types of classification structures

Broad and fined grained classifications can be linked by assuming a hierarchically-organized system of classes (Corbett & Fraser 1993; Dressler & Thornton 1996).

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In recent years, various efforts have been made towards inferring inflection class hierarchies automatically from paradigms (Brown & Hippisley 2012; Lee & Goldsmith 2013; Bonami 2014). While they use very different methodologies, most of these approaches converge on the use of tree shaped hierarchies (Figure 1b.). Network Morphology (Corbett & Fraser 1993; Brown & Hippisley 2012) uses richer structure through default inheritance and multiple inheritance of orthogonal properties, but does not allow for multiple inheritance in a single dimension (e.g. affixes).

In this paper, we argue that while "Inflection classes" usually refers to either partitions (Figure 1a.) or trees (Figure 1b.), these make simplifications which overlook numerous relations between lexemes and hide structural properties that are in fact pervasive. We show that semi-lattices (Figure 1c.), where one subclass may belong to more than one superclass, are more faithful models of inflectional systems. We use formal concept analysis (Ganter & Wille 1998) to automatically infer semi-lattices of inflection classes for the verbal systems of French, English, Modern Standard Arabic, European Portuguese, and Zenzontepec Chatino, and for the nominal system of Russian.

We compare these systems through canonical typology. To do so, we provide formal definitions of inflectional structure and precise quantitative measures of inflectional canonicity, which can be computed automatically from large inflected lexicon.

Inflection classes are usually taken as classes of lexemes or stems related by common affixes (Carstairs 1987; Carstairs-McCarthy 1991; Stump & Finkel 2013). However, alternations between stems also contribute to the expression of inflectional information. Segmentation in stems and affixes is useful to produce systems in constructive approaches (in the sense of Blevins (2006)), where the goal is to generate the forms from a minimal grammar. We adopt on the contrary the abstractive approach (Blevins 2006) and we attempt to account for all interesting generalizations. As a consequence, we take **inflectional behavior** to be relations between word-forms, or **alternation patterns**, rather than affixes (Bonami & Luís 2014; Bonami & Beniamine 2016).

In the first section, we present partition and tree-based accounts of inflection classes. Next, we motivate the need for multiple inheritance hierarchies and show that simpler models make problematic predictions. In a third section, we present Formal Concept Analysis, and how it can be used to infer a semi-lattice of classes. The last section discusses the properties of the inflection class lattices.

1 The structure of inflection class systems

Inflection class systems are often described as a partition of a few broad classes of lexemes which share some of their inflectional behavior. Partitions of inflection classes are used both in pedagogical grammars and in many descriptive accounts. They usually count only a few classes. They are, as Matthews (1991: p. 129) puts it, “classes of lexemes that go together in respect of some inflection”. This definition relies on the inflectional similarity between lexemes.

Corbett (1982) counts six nominal inflection classes (declensions) in Russian, which we illustrate in Table 1 by showing the full paradigm of one exemplar lexeme per class. We indicate frequencies based on counts in a lexicon of 1239 nouns constructed in collaboration with Dunstan Brown and described in more detail in Appendix 4 and Beniamine (2018).

Table 1: Six broad inflection classes of Russian in roman transliteration, according to Corbett (1982: p. 203).

lexeme	ZAKON	VINO	ŠKOLA	KOST’	PUT’	VREMJA
glose	‘law’	‘wine’	‘school’	‘bone’	‘way’	‘time’
frequency	874	96	428	112	1	6
NOM.SG	zakon	vino	škola	kost’	put’	vremja
ACC.SG	zakon	vino	školu	kost’	put’	vremja
GEN.SG	zakona	vina	školy	kosti	puti	vremeni
DAT.SG	zakonu	vinu	škole	kosti	puti	vremeni
INST.SG	zakonom	vinom	školoj	kost’ju	putem	vremenem
LOC.SG	zakone	vine	škole	kosti	puti	vremeni
NOM.PL	zakony	vina	školy	kosti	puti	vremena
ACC.PL	zakony	vina	školy	kosti	puti	vremena
GEN.PL	zakonov	vin	škol	kostej	putej	vremen
DAT.PL	zakonam	vinam	školam	kostjam	putjam	vremenam
INST.PL	zakonami	vinami	školami	kostjami	putjami	vremenami
LOC.PL	zakonax	vinax	školax	kostjax	putjax	vremenax

While it is usually thought that there is only one correct inventory of inflection classes in a given system, the number of classes is in fact often disputed, even in very well documented languages. Corbett (1982: p. 202) highlights such disagreements in the case of Russian nouns: “The reader not familiar with the literature will quite reasonably expect a straightforward account of the paradigms in Russian. Tradition answers three, some writers claim four, and more recently it has been suggested that only two paradigms are required". The situation of Russian Nouns is far from exceptional. One reason is that constructive and pedagogical analyses both usually strive for the shortest possible description. This leads to the merging of classes whenever possible, for example where distinct surface realizations can be abstracted away as allomorphy, or predicted using semantic or grammatical properties of the lexemes. For example, Corbett shows that most descriptions of the inflection classes of Russian nouns merge the classes of zakon and vino, and those of kost' and put'. The class of vremja can either be merged or separate from this last class. In a similar fashion, Plénat (1987) described a two-class analysis of the French verbal inflectional system, which is usually described as having three conjugations. To do so, he merges the second and third conjugation using abstract phonological representations. Blevins (2004) reports that the nominal system of Estonian has been described as having between 26 and 400 "paradigms", which can be merged in 6 to 12 inflection classes.

Going back to the the data presented in Table 1, we show by gray cells some similarities between classes for each cell. All the classes share realizations for the dative, instrumental and locative plural. The class zakon shares the same endings as the class vino for the genitive, instrumental and locative singular. The locative singular is also identical to that of škola. zakon and škola also share the same endings in the nominative and accusative plural, while vino and škola both present no affixes in the genitive plural. The nominative and accusative singular of zakon, like those of kost' and put', show no affixes on the stem, etc. To these similarities in terms of endings or affixes, we could add similarities in terms of alternations, such as syncretisms: for example, the classes zakon, vino, kost', put' and vremja (but not škola) all present a syncretism between nominative and accusative singular. All these lexemes share a syncretism between the nominative and accusative plural.

A look at the Russian lexicon described in Appendix 4 shows that the behavior of lexemes inside each class is less homogeneous than suggested by the table of exemplars. While all the exemplars shown above are inanimate and present the accusative-nominative syncretism, we found 163 lexemes of the classes zakon, 8 of the class vino, 47 of the class škola and 6 of the class kost' which rather present an accusative-genitive syncretism, typical of animate nouns (see Corbett & Fraser 1993: p.129). Moreover 76 lexemes of the class zakon, 3 of the class vino and 6 of the class škola end in -ej rather than -ov or the bare stem.

Since similarity is gradient, it is difficult to determine how similar lexeme's behavior need to be to belong to the same class. Recent works in computational linguistics have attempted to decide on the best partition using minimal description length, either by comparing hand-written analysis (Walther & Sagot 2011) or by generating the analysis automatically from the data (Beniamine, Bonami & Sagot 2017). But even when selected very rigorously, the resulting partitions are simplifications. They can be useful as pedagogical tools, or as compact constructive descriptions, but they do not account for all similarities between classes, nor for the internal variation in each class.

At the other end of the descriptive spectrum, various studies take inflection classes as very fine grained partitions, where each distinction in inflectional behavior warrants a separate class. IC membership is then defined in terms of identity. Aronoff (1994: p. 64) defines an IC as "a set of lexemes whose members each select the same set of inflectional realizations". Carstairs-McCarthy (1994: p. 739) provides two definitions of a paradigm:

> (1) PARADIGM$_1$: the set of combinations of morphosyntactic properties or features (or the set of 'cells') realized by inflected forms of words (or lexemes) in a given word-class (or major category or lexeme-class) in a given language.
> (2) PARADIGM$_2$: the set of inflectional realizations expressing a paradigm$_1$ for a given word (or lexeme) in a given language.

Based on these definitions, he offers a very similar definition of inflection classes: "a set of words (lexemes) displaying the same paradigm$_2$ in a given language", where paradigm$_2$ is defined as. Applied to realistic datasets, these definitions yield a high number of classes, many of which are often very small. Stump & Finkel (2013) count 72 inflection classes for French verbs, while Bonami (2014); Beniamine, Bonami & Sagot (2017); Beniamine (2018) count up to 97 classes[1]. For Russian nouns, Beniamine (2018) counts 159 IC based on identity of surface segmental inflectional behavior (not counting stress patterns). While by definition, these classes do not show any internal heterogeneity, enumerating them does not account for any similarities across classes.

Descriptive grammars often make use of explicit or implicit tree-shaped hierarchies when they provide several granularity levels. For example, the French pedagogical grammar Bescherelle (Arrivé 2012) describes three inflection classes,

[1] While they all base their computations on the Flexique lexicon (Bonami, Caron & Plancq 2014), differences across accounts are due both to different methodologies and to corrections that have been made in the lexicon since its publication.

each exemplified by numerous verbal exemplars (one per page) and finer variations in footnotes. These can be interpreted as a three level hierarchy. Campbell (2011) describes the inflection classes in Zenzontepec Chatino, an Oto-Manguean language spoken in Oaxaca, by a three level hierarchy presented in Figure 2. Zenzontepec Chatino expressed inflection through prefixes and has only four paradigm cells: potential, habitual, progressive and completive. Figure 2 shows common prefixes for each node of the hierarchy. The notation "[lam]" marks the laminalization of initial [t] in class Bt. Campbell (2011) shows identical underlying prefixes for classes Au and Ac, but they differ on the surface. Class Bc presents a stem initial alternation between y- and ch-. Since class C2 presents several distinct affixes, it could be further divided in two distinct classes. The first level of Campbell (2011)'s classification is not based on similarity alone, but inherits from Kaufman (1989)'s description of Zapotec inflection classes.

Dressler & Thornton (1996); Kilani-Schoch & Dressler (2005); Dressler et al. (2008) call macro-classes the broad ICs based on similarity and micro-classes the fine-grained inflection classes based on identity of inflectional behavior. They link both in tree-shaped hierarchies, in which any node can be seen as an inflection class. Micro-classes form the leaves of the hierarchy, while macro-classes form the first level below the root. Any number of intermediate classes can appear in between. In Kilani-Schoch & Dressler (2005)'s approach to French, the macro-classes are not based on similarity alone, but instead they constitute a bipartition between productive and unproductive patterns. Each inflection class is motivated by common inflectional patterns, written as implicative statements which the authors call "paradigm structure conditions". These conditions are inherited by default.

In Network morphology (Corbett & Fraser 1993; Brown & Hippisley 2012), inflection classes are also represented by a tree-shaped default inheritance hierarchy. The analyses are constructive: couched in the DATR formalism, each node specifies affixal rules. The grammar is designed to generate surface forms. Default inheritance has two main advantages. First, it allows for more compact representations by limiting repetitions and the overall number of nodes in the hierarchy. Second, it gives a natural status the notion of regularity: a node which rewrites a default is exceptional relatively to the ancestor which stipulated the default rule.

Going back to Russian nouns, Brown (1998) count four main inflection classes which can be mapped to the first four declensions described by Corbett (1982): I is the class of ZAKON, II is that of ŠKOLA, III that of KOST' and IV that of VINO. Brown (1998) argues in favor of the hierarchical structure which we summarize

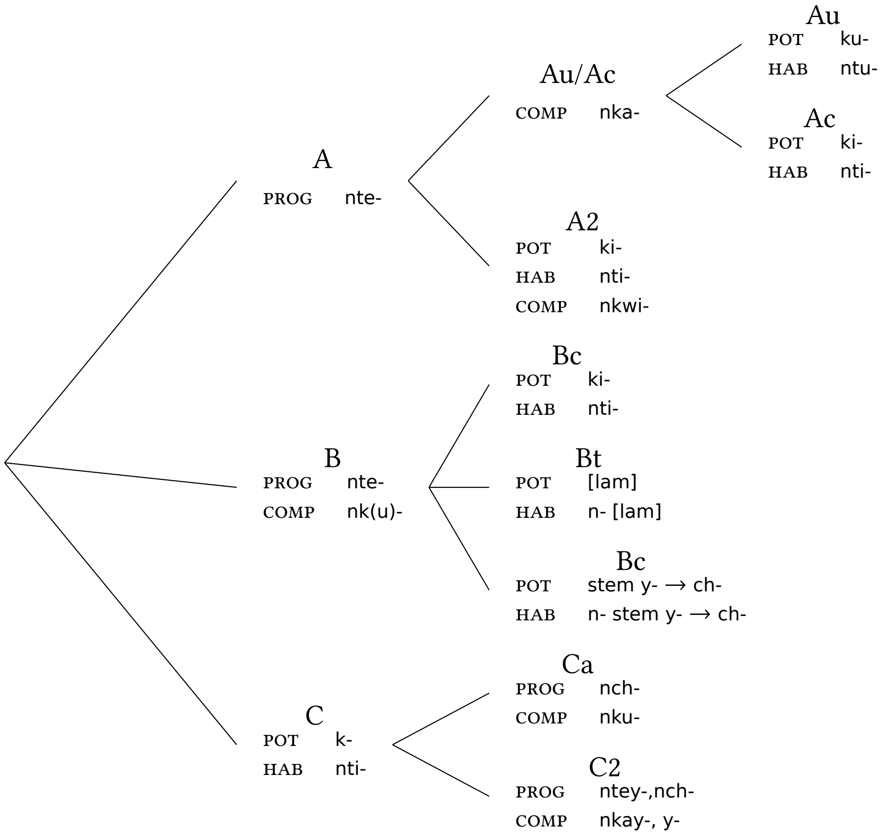


Figure 2: Inflection class tree in Zenzontepec Chatino verbs according to Campbell (2011: p. 229)

in Figure 3. In the inflectional tree, the leaves N_I to N_IV stand for each of the four inflection classes. The root is the node MOR_NOMINAL, which also spans adjectives (which we will ignore for the purpose of this paper). It defines common properties between nouns and adjectives, as well as two default values: a zero affix in the nominative singular, and an -i ending in the nominative plural. The term *evaluation* denotes the usage of a realization function which takes as input morphological properties of a lexeme and can assign distinct values to lexemes belonging to the same class. The node MOR_NOM specifies a thematic vowel characteristic of all nouns, a default affixal value for the locative singular, and a default syncretism between dative and locative singular. There is only one intermediate node, N_O, which manifests properties shared between classes I et IV.

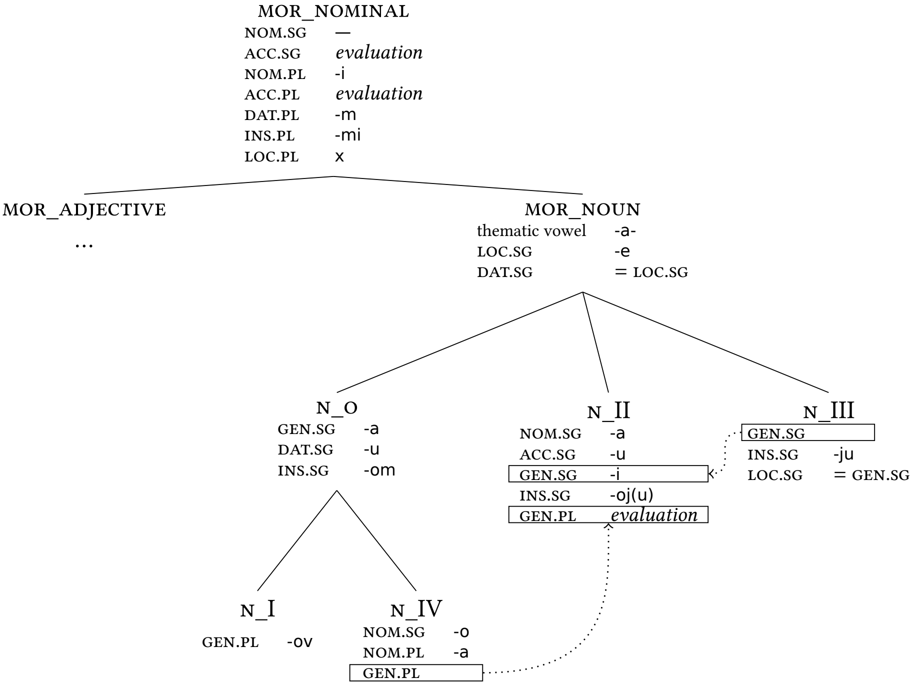


Figure 3: DATR hierarchy for Russian nouns according to Brown (1998: Theory B, p. 128 et seq.)

In Brown (1998)'s theory, some commonalities between classes are not modeled through the tree structure itself, but by direct references across classes' paradigm cells. We write these references by dotted arcs between framed cells. For example, the genitive plural of class IV is formed by using the evaluation functions of the genitive plural in class II. The need for this second mechanism highlights the inadequacy of a tree structure to express all similarities between inflection classes. In addition, while default inheritance is useful to produce a compact hierarchy, it hides the exact span of the default rules. In the following section, we show how a richer hierarchy can account more naturally for inflection class structure in an abstractive approach.

2 Non canonical systems as inflection class lattices

In the previous section, we showed that partitions and tree structures are often used to describe inflectional systems whose similarity structure is more complex than these descriptive devices can account for. It is however conceivable that some inflectional systems do conform to the structure of either a partition or a tree.

Corbett (2009) chooses this particular ideal structure as a canonical point of comparison for typological investigation. According to him, a canonical inflection class system follows the principle of distinctiveness, which can be evaluated through four criteria:

> PRINCIPLE I (distinctiveness): Canonical inflection classes are fully comparable and are distinguished as clearly as is possible. [...]

criterion 1 In the canonical situation, forms differ as consistently as possible across inflectional classes, cell by cell.

criterion 2 Canonical inflectional classes realize the same morphosyntactic or morphosemantic distinctions (they are of the same structure).

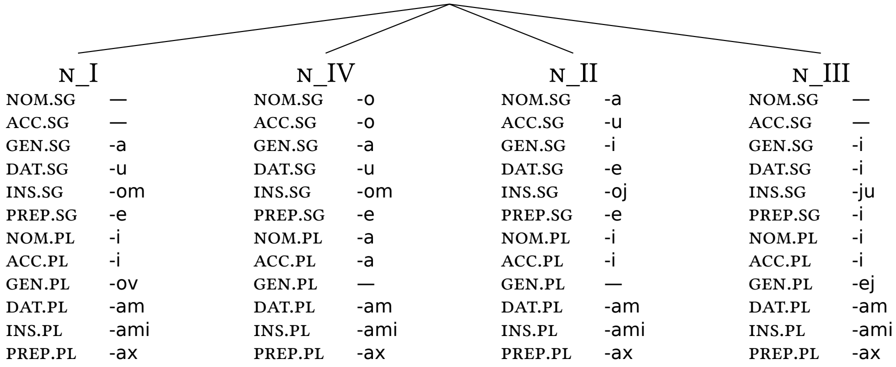
criterion 3 Within a canonical inflectional class each member behaves identically.

criterion 4 Within a canonical inflectional class each paradigm cell is of equal status.

From these criteria, it follows that in a canonical system, there are no similarities between classes. If two classes were to have a common exponent or alternation pattern, they would violate criterion 1. Moreover, the cells affected by common patterns would then be less predictive of the inflection classes than other cells, which violates criterion 4. According to criterion 2, a canonical system of inflection classes can have only one form per paradigm cell and lexeme. Defective lexemes, which lack forms for certain cells, and overabundant lexemes, which have more than one possible form for certain cells, violate criterion 2. Finally criterion 3 means that all classes are micro-classes: they are based on identity. In a canonical system, micro- and macro-classes coincide. The system then truly has the shape of a partition (or a one-level tree, with classes as leaves and the whole system as root).

If real systems mostly conformed to the canonical ideal – which is not usually expected – then it could be adequate to model them using partitions. If however, non-canonicity is the norm, then more expressive models would be required. Since partitions and trees make the assumption of a certain degree of canonicity, these models are not suited to evaluate a system's position in the canonical space.

Figure 4 shows the same four inflection classes of Russian nouns as in Figure 3, now arranged as a partition, with each class characterized by affixes. While the shape of this classification is that of a partition, it is obvious from the numerous repetitions that it is not the structure of the data. The use of a partition masks the system's non-canonicity.



N_I		N_IV		N_II		N_III	
NOM.SG	—	NOM.SG	-o	NOM.SG	-a	NOM.SG	—
ACC.SG	—	ACC.SG	-o	ACC.SG	-u	ACC.SG	—
GEN.SG	-a	GEN.SG	-a	GEN.SG	-i	GEN.SG	-i
DAT.SG	-u	DAT.SG	-u	DAT.SG	-e	DAT.SG	-i
INS.SG	-om	INS.SG	-om	INS.SG	-oj	INS.SG	-ju
PREP.SG	-e	PREP.SG	-e	PREP.SG	-e	PREP.SG	-i
NOM.PL	-i	NOM.PL	-a	NOM.PL	-i	NOM.PL	-i
ACC.PL	-i	ACC.PL	-a	ACC.PL	-i	ACC.PL	-i
GEN.PL	-ov	GEN.PL	—	GEN.PL	—	GEN.PL	-ej
DAT.PL	-am	DAT.PL	-am	DAT.PL	-am	DAT.PL	-am
INS.PL	-ami	INS.PL	-ami	INS.PL	-ami	INS.PL	-ami
PREP.PL	-ax	PREP.PL	-ax	PREP.PL	-ax	PREP.PL	-ax

Figure 4: Partition of four russian inflection classes

We have seen in Figure 3 that a tree structure, which could be seen as an intermediate level of canonicity, can also be insufficient to express all the similarities between these inflection classes. In Figure 5, we offer an analysis which accounts for each point of similarity between the four classes from Figure 4. This analysis does not allow any other inheritance mechanism than the hierarchy itself: as a consequence, it does not contain defaults, rules of referral, or evaluation functions.[2]

Contrarily to a tree, the hierarchy in Figure 5 displays multiple inheritance. For example, class I has two parents. On one side, it inherits the absence of affix in the nominative and accusative singular, and on the other side it inherits values for its genitive, dative and instrumental singular affixes. This structure is a lattice, as is the type hierarchy in HPSG (Flickinger 1987; Pollard & Sag 1994; Ginzburg

[2] In the interest of legibility, we take classes I to IV to be micro-classes, and exclude some lexemes which Brown (1998) account for using evaluation functions. The hierarchy can however be extended to account for all micro-classes of a system. For the same reason, we ignore adjectives in this example.

& Sag 2000), or phonological features hierarchies (Chomsky & Halle 1968; Frisch 1997). Since inflection classes can be seen as "classes of lexemes that share similar morphological contrasts" (Brown & Hippisley 2012: p. 4), we call an inflection class any node of such a hierarchy, and not only its leaves. In consequence, one lexeme can belong to many inflection classes.

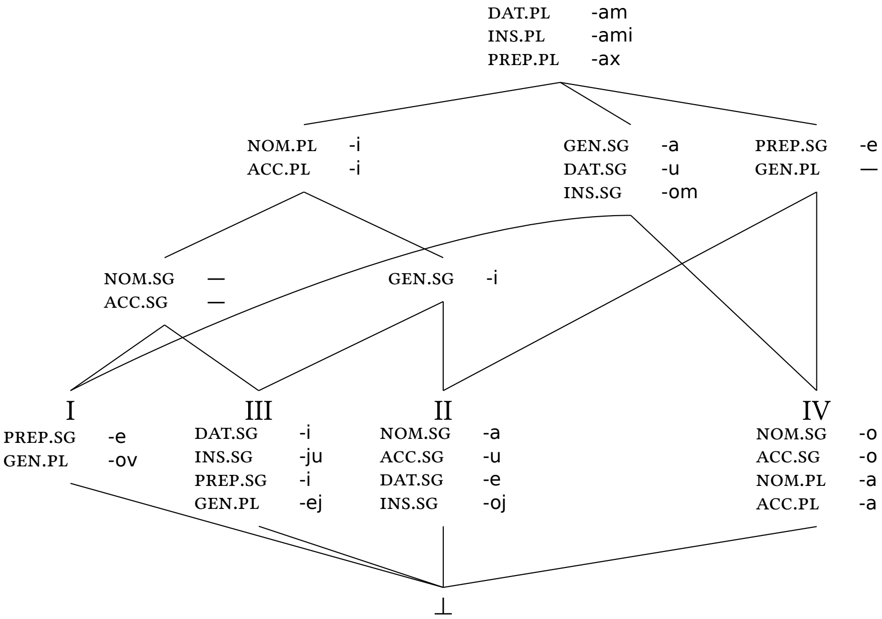


Figure 5: Lattice of four russian inflection classes

In the hierarchy from Figure 5, each intermediate node represents a similarity point between lower nodes, and all similarities between lexemes are expressed in this way. One can read all the information relating to a class by going through each of its ancestors. Information specified on the leaves is entirely distinctive: it is specific to each micro-class. Values indicated on the root are on the contrary common to all inflection classes. In the hierarchy, higher nodes hold more general information than lower nodes: their value is less specified and they encompass more classes. Classes are ordered by increasing generality in the lattice.

Given two nodes in the hierarchy, one can read exactly what classes have them in common by searching their greatest lower bound, also called *meet*, which is their highest common child. There is only one such child. For example, the node {NOM.PL -i, ACC.PL -i} and the node {PREP.SG -e, GEN.PL -} have the class II for greatest lower bound. The lowest node in the hierarchy, or infimum, noted ⊥, is the *meet* between any pair of the leaves, because no lexeme can belong to more than one of these micro-classes. Symmetrically, one can find the common information between any two classes by searching for their least upper bound, or *join*. Since the infimum is always present and never brings any relevant information, we will sometimes omit it.

This hierarchy displays precisely what distinguishes this system from the canonical situation. While canonical inflection classes have only micro-classes and a supremum (root) as is the case in Figure 4, the structure in Figure 5 has five more intermediate classes. A hierarchy of canonical inflection classes has a depth of 1, but the lattice from Figure 5 has a depth of 3 (the longest path from the root to a micro-class follows three edges). Finally, while the canonical situation shows only simple inheritance, classes in this hierarchy have on average 1.4 direct parents.

We saw that a partition model makes the prediction that the classes are canonical, which isn't the case of the partial systems we discussed. A tree structure allows some sharing across micro-classes, but still makes a prediction on their canonicity. It assumes that while classes can share some properties, there is no heteroclite sharing. *Heteroclisis* is usually taken to occur when the paradigm of a small inflection class is split in such a way that it follows two or more separate distinct inflection classes (Corbett 2009). The term can be extended in order to describe any class which displays multiple inheritance. Modeling inflection class systems as lattices will allow us to observe the amount of heteroclite sharing and quantify their canonicity.

3 Inferring inflection class lattices with formal concept analysis

To automatically produce an inflectional lattice, we use Formal Concept Analysis (Ganter & Wille 1998), hereafter FCA. This mathematical formalism allows us to study all interesting relationships between objects (in our case lexemes, or micro-classes), and attributes instantiated by these lexemes, by studying sets of objects and properties ordered in a *conceptual hierarchy*. This section describes the basics of FCA, illustrated on a few sub-paradigms of English verbs shown in Table 2.

In the previous sections, we took inflectional attributes to be affixes. However, using affixes to automatically assess similarity of inflectional behavior is problematic (Beniamine 2018): first, they do not account for all similarities between paradigms (Beniamine, Bonami & Sagot 2017), second, ignoring stem al-

Table 2: Some sub-paradigms of english verbs.

lexeme	PST	PST.PART	PRS
DRIVE	/drə·ʊv/	/drɪvn̩/	/dra·ɪv/
RIDE	/rəʊd/	/rɪdn̩/	/ra·ɪd/
BITE	/bɪt/	/bɪtn̩/	/ba·ɪt/
FORGET	/fəgɒt/	/fəgɒtn̩/	/fəgɛt/

ternations excludes a large number of relevant inflectional properties (Bonami & Beniamine 2016). Last, but not least, there is no consensual method for segmenting wordforms into affixes (Spencer 2012). For these reasons, we rely rather on alternation patterns (Bonami & Luís 2014; Bonami & Beniamine 2016), which can be automatically inferred from raw forms in a language-agnostic way (Beniamine 2018; 2017). The program we use finds alternation patterns from a fully inflected lexicon structured as a paradigm table (as in Table 2). Forms are transcribed in phonemic notation, and the lexicon is accompanied by a decomposition of each phonemes in minimal features (see Appendix 4). Both the structure of the paradigm table and the transcription constitute idealizations.

We show in Table 3 the alternation patterns deduced from pairwise alternations from Table 2. For example, the alternation between /fəgɛt/ (PRS) and /fəgɒt/ (PST) follows the bidirectional alternation pattern _ɛ_ ⇌ _ɒ_, where "_" indicates the presence of constant material in the form.[3] We note ϵ the empty string.

Table 3: Alternation patterns for the subparadigms from Table 2.

lexeme	PST.PART ⇌ PRS	PST.PART ⇌ PST	PRS ⇌ PST
RIDE	_ɪ_n̩ ⇌ _a·ɪ_	_ɪ_n̩ ⇌ _ə·ʊ_	_a·ɪ_ ⇌ _ə·ʊ_
DRIVE	_ɪ_n̩ ⇌ _a·ɪ_	_ɪ_n̩ ⇌ _ə·ʊ_	_a·ɪ_ ⇌ _ə·ʊ_
BITE	_ɪ_n̩ ⇌ _a·ɪ_	_n̩ ⇌ _ϵ	_a·ɪ_ ⇌ _ɪ_
FORGET	_ɒ_n̩ ⇌ _ɛ_ϵ	_n̩ ⇌ _ϵ	_ɛ_ ⇌ _ɒ_

[3] We report here a simplified view of alternation patterns, specifying only the alternating material as well as its position in the word. The program we use (Beniamine 2017; 2018) also extracts a detailed set of phonotactic constraints on the context of the changes. We omit it here in all examples for simplicity.

Table 3 defines a relationship between lexemes and alternation patterns. It can be written as an incidence matrix, that is a cross table where objects are indicated in rows, attributes in columns, and where a cross in a cell indicates that the object in this row instantiates the property in this column. Such a table is called a *formal context*. Table 4 shows the context for the subparadigms of English verbs from Table 2. We take objects to be lexemes, and attributes to be combinations of a pair of cell and an alternation pattern.

Table 4: Formal context for Table 3.

	PST.PART⇌PRS		PST ⇌ PRS			PST.PART ⇌ PST	
	_ɪ_n̩ ⇌ _aɪ_	_ɒ_n̩ ⇌ _ɛ_ϵ	_aɪ_ ⇌ _əʊ_	_aɪ_ ⇌ _ɪ_	_ɛ_ ⇌ _ɒ_	_ɪ_n̩ ⇌ _əʊ_	_n̩ ⇌ _ϵ
DRIVE	×		×			×	
RIDE	×		×			×	
BITE	×			×			×
FORGET		×			×		×

A *formal context* is a triplet $\langle X, Y, I\rangle$, where X and Y and nonempty sets, and I is a binary incidence relation between X (objects, in line) and Y (attributes, in column): $I \subseteq X \times Y$. For all objects $x \in X$ and all attributes $y \in Y$:

- $\langle x, y\rangle \in I$ indicates that the object x has the attribute y,
- $\langle x, y\rangle \notin I$ indicates that x does not have y.

In the context table $\langle X, Y, I\rangle$, there is a cross at coordinates i, j if and only if $\langle x_i, y_i\rangle \in I$. Ganter & Wille (1998) write $\langle x, y\rangle \in I$ as xIy.

For any subset of objects $A \subset X$, we are interested in the attributes they have in common, and for any subset of attributes $B \subset Y$, we are interested in the objects which instantiate them. We define two operators, "↑" and "↓" (Bělohlávek 2009: p. 6-7), such that:[4]

[4] This notation is that of Bělohlávek (2009), while Ganter & Wille (1998) note both by $'$, and the sets $A \uparrow$ and $B \downarrow$ respectively by A' and B'. We prefer Bělohlávek's (2009) more explicit convention.

- The operator ↑ maps objects (subsets of X) to attributes (subsets of Y). $A\uparrow$ is defined as the subset of all attributes shared by the objects in A:

$$\uparrow: 2^X \rightarrow 2^Y \text{ and } A \uparrow= \{y \in Y | \text{ for each } x \in A : xIy\}$$

- The operator ↓ maps attributes (subsets of Y) to objects (subsets of X). $B\downarrow$ is defined as the subset of all objects which share all attributes in B:

$$\downarrow: 2^Y \rightarrow 2^X \text{ and } B \downarrow= \{x \in X | \text{ for each } y \in B : \langle xIy\}$$

If objects in A have no common attribute, then $A \uparrow= \emptyset$. Similarly, if no object shares all the properties from B, then $B \downarrow= \emptyset$. Consequently, $\emptyset \uparrow= Y$ and $\emptyset \downarrow= X$.

In our example, we have[5]:

(1) {RIDE, DRIVE}↑ = {_ɪ_n̩ ⇌ _aˑɪ_ , _aˑɪ_ ⇌ _əˑʊ_, _ɪ_n̩ ⇌ _əˑʊ_}

(2) {_ɪ_n̩ ⇌ _aˑɪ_, _aˑɪ_ ⇌ _əˑʊ_, _ɪ_n̩ ⇌ _əˑʊ_}↓ = {DRIVE, RIDE}

(3) {_ɪ_n̩ ⇌ _aˑɪ_}↓ = {DRIVE, RIDE}

(4) {_aˑɪ_ ⇌ _ɪ_, ɛ ⇌ ɒ}↓ = ∅

These equalities can be read directly in Table 4. The lexemes DRIVE and RIDE share all of their attributes (1). The three patterns shared by them are only shared by them (2). The pattern _ɪ_n̩ ⇌ _aˑɪ_ is also shared by only DRIVE and RIDE (3). Finally, the operator ↓ applied to concurrent contradictory pattern for PST ⇌ PRS, produces the empty set (4) unless there are overabundant lexemes instantiating these patterns.

Using these operators, we can define a *formal concept.* A formal concept in the context $\langle X, Y, I\rangle$ is a pair $\langle A, B\rangle$ of a set of objects $A \subseteq X$ called the *extension* of the concept, and a set of attributes $B \subseteq Y$ called the *intension* of the concept, such that $A \uparrow= B$ and $B \downarrow= A$. In other words, the objects from A have in common exactly the attributes from B, no more, no less. Reciprocally, the attributes from B are common to all objects in A, no more, no less.

For example, ⟨{DRIVE,RIDE}, {_ɪ_n̩ ⇌ _aˑɪ_, _aˑɪ_ ⇌ _əˑʊ_, _ɪ_n̩ ⇌ _əˑʊ_ }⟩ is a formal concept, because we have both (1) and (2). However, ⟨{DRIVE,RIDE}, {_ɪ_n̩ ⇌ _aˑɪ_

[5] In all examples below and in Figures 6 and 7, we do not repeat morphosyntactic attributes for the alternation patterns. This is a shortcut, as our attributes are actually combinations of a pair of cell and an alternation patterns. In our small example, where only seven patterns are considered, this omission does not lead to ambiguity. However, due to syncretism, this would not be the case for most real systems.

}⟩ is not a formal concept, because despite (3), the opposite isn't true, as {_ɪ_n̩ ⇌ _aˑɪ_ is only a subset of {RIDE, DRIVE}↑ (1).

From the incidence table, it is possible to produce the list of all the formal concepts. Examples (5) through (11) list all the concepts present in Table 4:

(5) ⟨ ∅, {_ɒ_n̩ ⇌ _ɛ_ϵ, _ɪ_n̩ ⇌ _aˑɪ_, _n̩ ⇌ _ϵ, _ɪ_n̩ ⇌ _əˑʊ_, _aˑɪ_ ⇌ _əˑʊ_, _aˑɪ_ ⇌ _ɪ_, _ɛ_ ⇌ _ɒ_} ⟩

(6) ⟨ {BITE}, {_ɪ_n̩ ⇌ _aˑɪ_, _n̩ ⇌ _ϵ, _aˑɪ_ ⇌ _ɪ_} ⟩

(7) ⟨ {FORGET}, {_ɒ_n̩ ⇌ _ɛ_ϵ, _n̩ ⇌ _ϵ, _ɛ_ ⇌ _ɒ_} ⟩

(8) ⟨ {RIDE, DRIVE}, {_ɪ_n̩ ⇌ _aˑɪ_, _ɪ_n̩ ⇌ _əˑʊ_, _aˑɪ_ ⇌ _əˑʊ_} ⟩

(9) ⟨ {BITE, FORGET}, {_n̩ ⇌ _ϵ} ⟩

(10) ⟨ {RIDE, DRIVE, BITE}, {_ɪ_n̩ ⇌ _aˑɪ_} ⟩

(11) ⟨ {RIDE, DRIVE, BITE, FORGET}, ∅ ⟩

We noted, when observing the lattice in Figure 5, that classes were ordered by specificity. Concepts can also be ordered according to their specificity. Given two concepts $\langle A_1, B_1 \rangle$ and $\langle A_2, B_2 \rangle$ in $\langle X, Y, I \rangle$, $\langle A_1, B_1 \rangle$ is more specific than $\langle A_2, B_2 \rangle$ if and only if A_1 is a subset of A_2, which entails that B_2 is a subset of B_1. We call $\langle A_1, B_1 \rangle$ a subconcept of $\langle A_2, B_2 \rangle$:

$$\begin{aligned}\langle A_1, B_1 \rangle \leq \langle A_2, B_2 \rangle &\iff A_1 \subseteq A_2 \\ &\iff B_2 \subseteq B_1\end{aligned}$$

In other words, the subconcept contains only some of the objects (lexemes) from the more general concept, but more attributes (patterns). For example, the concept in example (8) is a subconcept of the concept in example 10. The subconcept has one less lexeme and two more patterns.

If $\langle A_1, B_1 \rangle \leq \langle A_2, B_2 \rangle$ and there are no concepts $\langle A_i, B_i \rangle$ in $\langle X, Y, I \rangle$ such that $\langle A_1, B_1 \rangle \leq \langle A_i, B_i \rangle \leq \langle A_2, B_2 \rangle$, then $\langle A_1, B_1 \rangle$ is an immediate lower neighbor of $\langle A_2, B_2 \rangle$, which we write: $\langle A_1, B_1 \rangle \prec \langle A_2, B_2 \rangle$.

The collection of all formal concepts of a context $\langle X, Y, I \rangle$, together with the order relation $\leq$, form the *concept lattice* of $\langle X, Y, I \rangle$, written $\mathcal{B}\langle X, Y, I \rangle$. A finite ordered set can be represented by a Hasse diagram in which each element of the set is a node in a hierarchical structure. If an element is a subconcept of another, it is written lower in the diagram. Edges link immediate neighbors. For any pair of concepts c_1, c_2 in $\langle X, Y, I \rangle$, we have $c_1 \leq c_2$ if c_2 can be reached from c_1 by an ascending path.

Figure 6 shows the hierarchical representation of the context lattice from Table 4 as a Hasse diagram. Each node is annotated by its concept.

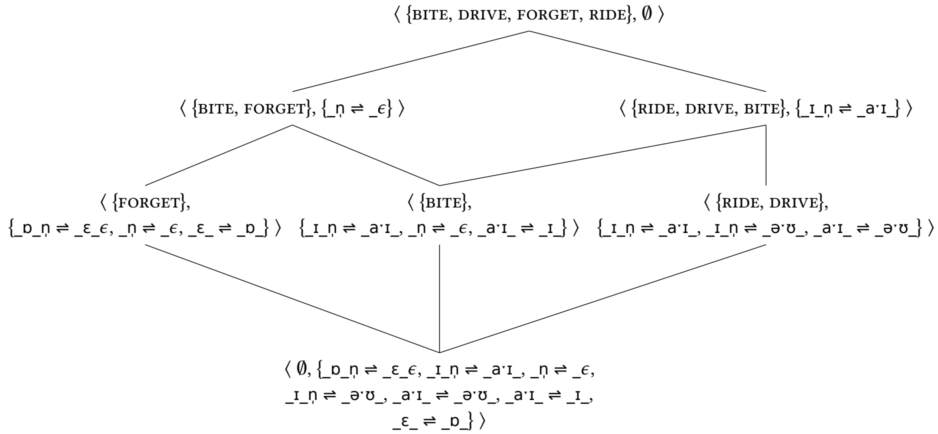

⟨ {BITE, DRIVE, FORGET, RIDE}, ∅ ⟩
⟨ {BITE, FORGET}, {_n̩ ⇌ _ϵ} ⟩
⟨ {RIDE, DRIVE, BITE}, {_ɪ_n̩ ⇌ _aˑɪ_} ⟩
⟨ {FORGET}, {_ɒ_n̩ ⇌ _ɛ_ϵ, _n̩ ⇌ _ϵ, _ɛ_ ⇌ _ɒ_} ⟩
⟨ {BITE}, {_ɪ_n̩ ⇌ _aˑɪ_, _n̩ ⇌ _ϵ, _aˑɪ_ ⇌ _ɪ_} ⟩
⟨ {RIDE, DRIVE}, {_ɪ_n̩ ⇌ _aˑɪ_, _ɪ_n̩ ⇌ _əˑʊ_, _aˑɪ_ ⇌ _əˑʊ_} ⟩
⟨ ∅, {_ɒ_n̩ ⇌ _ɛ_ϵ, _ɪ_n̩ ⇌ _aˑɪ_, _n̩ ⇌ _ϵ, _ɪ_n̩ ⇌ _əˑʊ_, _aˑɪ_ ⇌ _əˑʊ_, _aˑɪ_ ⇌ _ɪ_, _ɛ_ ⇌ _ɒ_} ⟩

Figure 6: Concept lattice for the context in Figure 4

However this notation is redundant. It is not necessary to repeat on higher nodes objects that have been defined by lower concepts, as they can be deduced from the hierarchical structure. Symmetrically, it is not necessary to repeat on lower nodes attributes that have been defined by higher concepts. The reduced notation only writes objects and attributes in the structure on the concepts which define them. Figure 7 shows the same lattice as 6, in reduced notation. Concept lattices written in reduced notation can be read as monotonous multiple inheritance hierarchies. The resulting hierarchy is unique. It is entirely deduced from the context table and there are no possible alternative structures which fit with the above definitions.

4 Properties of inflection class lattices

In this section, we apply the methodology described in the previous section to a few inflectional systems, and investigate the similarity structure across their paradigms. We built inflection class lattices for the verbal systems of Modern Standard Arabic, English, French, European Portuguese, and Zenzontepec Chatino, and for the nominal system of Russian. These languages are chosen for their variety and the availability of the computational resources needed for a quantitative investigation. The selection does not constitute a typologically representative sample, but it illustrates a variety of inflectional strategies.

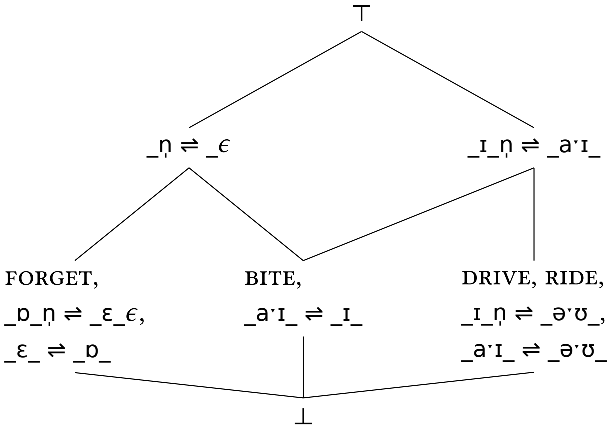


Figure 7: Concept lattice for the context in Figure 4, reduced notation

For a description of the input datasets, see appendix 4. As a first step before inferring inflection class lattices, we compute alternation patterns between all pairs of cells automatically from surface forms using the *Qumin* toolkit (Beniamine 2017; 2018).

Russian declensions have been described as the conjuction of two separate systems: one affixal and one made of stress alternations (Brown & Hippisley 2012). Similarly, Campbell (2016) described Zenzontepec Chatino inflection as made of "two orthogonal layers, the prefixal system and the tone alternation system, simultaneously at play". Because alternation patterns describe change in a holistic way, inferring alternation patterns on whole forms in these datasets leads to a multitude of rare patterns which represent the many possible intersections of two more general phenomena, one on each dimension. As a solution, we divide the datasets in two parts, then join the two resulting tables before inferring the classifications. For Russian, we create one table containing solely phonological segments, and one containing solely stress information. For Zenzontepec Chatino, we create separate segmental and tonal tables. Ideally, we would like to be able to make such decisions automatically, but this enterprise is left to future work. For more discussion on the subject, see Beniamine (2018).

We define micro-classes as the partition of lexemes which instantiate exactly the same alternation patterns for all pairs of cells: these are identical lines in the alternation pattern table. We keep only one entry representative of each micro-class, which we call the *exemplar* lexeme. The choice of the exemplar is arbitrary. To build inflectional context tables, we take objects to be micro-class exemplars, and attributes to be combinations of a pair of cell and alternation pattern. The resulting contexts are very large. We use the python library *concepts* (Bank 2016) to generate all concepts from the context table, and order them by specificity.

We obtain very large lattices. As an example, Figure 8 shows the overall structure of French and English lattices. We labelled objects on the structure next to the concept which defines them. We did not label alternation patterns, for legibility purposes.These examples are typical of the situation for all observed languages: the structures are by far too large for manual exploration and multiple inheritance is pervasive.

This fact in itself invalidate the hypothesis according to which real inflectional system could be appropriately described as either partitions or trees. Computing the whole similarity structure now allows us to quantify precisely how far from the canon these systems fall. We operationalize three measures described in section 2:

- **Number of concepts**: in the canonical situation, if a lattice has b leaves, there are exactly $b + 1$ concepts in the system (ignoring the infimum), the only other concept being the supremum. The higher the number of concepts, the more an inflectional system violates criterion 1 (distinctivity).

- **Depth of the hierarchy**: In the canonical situation, the longest path (and in fact, all paths) from the root to a leaf passes through only one edge. Evaluating the depth of the hierarchy gives us information regarding the type of sharing between classes. A high hierarchy is organized in successive classes and subclasses, and has more implicative structure than a shallower hierarchy, as in the lattice, any concept implies its ancestors. The higher the hierarchy, the more it violates criterion 4 (flat implicative structure).

- **Mean degree**: A canonical inflection class hierarchy is a one level tree. A multi level tree is a minor deviation from the canon. In a tree, the mean in-degree is 1 (ignoring the root, which has no incoming edges). Mean degree indicates the amount of multiple inheritance in the hierarchy. The higher the mean degree, the more the structure violates criterion 1 through heteroclite sharing.

Table 5 shows these measures for each system, as well as the number of lexemes in the dataset and the number of microclasses based on inflectional patterns. It is notable that the number of concepts found in each dataset is often comparable to the number of lexemes. In Arabic, there are 10 times more concepts than lexemes, and in Russian, there are 35 times more concepts than lexemes.

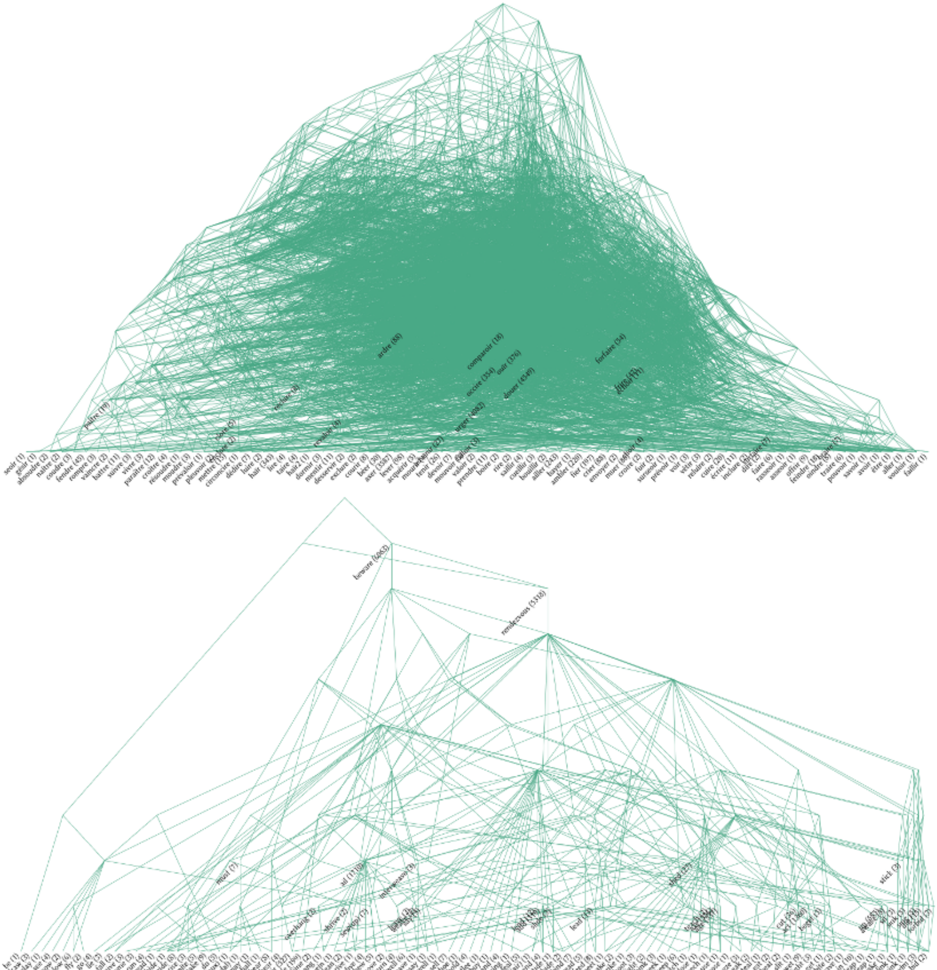


Figure 8: Inflection class lattices for French (top) and English (bottom) verbs.

In French and Zenzontepec Chatino, the number of concepts and lexemes are of the same order. In English and Portuguese, there are less concepts than lexemes, though the number of concepts is still high. This shows an important deviation from the conception according to which inflection classes provide a summary of inflectional behaviors.

Table 5: Canonicity measures of inflection class lattices based on alternation patterns

	Lexemes	Micro-classes	Leaves	Depth	Degree	Concepts
Arabic	1018	367	302	33	3.65	10125
English	6064	118	88	11	1.91	244
French	5249	97	77	27	3.96	4845
Russian	1529	226	208	26	5.19	53858
Portuguese	1996	60	60	21	2.79	677
Zenzontepec Chatino	324	99	98	8	2.65	524

The mean in-degree in all systems is close to or higher than 2, indicating that heteroclisis is the general case. Depth and number of concepts are always much higher than in the canonical situation, although it is difficult to compare these raw numbers from a dataset to another, given that the number of leaves varies.

To be able to compare these values across datasets, we calculate a relative depth and a relative number of concepts (or density). Given a lattice with b leaves and a depth of h, we normalize this depth by the maximal possible depth over b leaves, which is $b-1$ (ignoring the infimum):

$$\text{relative depth}(\mathcal{B}\langle X,Y,I\rangle\ ) = \frac{h}{b-1} \tag{1.1}$$

The maximal depth $b-1$ corresponds to the least possible canonical situation, where the lattice is the power set over the b leaves. In that case, there are $n = 2^b - 1$ concepts. We thus normalize the number of concepts in the lattice by this maximal value, and call the resulting measure *density*. If a lattice $\mathcal{B}\langle X,Y,I\rangle$ has n concepts over b leaves, then its density is:

$$\text{density}(\mathcal{B}\langle X,Y,I\rangle\ ) = \frac{n}{2^b-1} \tag{1.2}$$

Figure 9 shows these values for each system. The growth of 2^b is such that compared to the maximum non-canonicity conceivable, our lattices have very few nodes, resulting in very low densities (all below 10^{-10}), even when the absolute number of nodes is high. The differences in density in Figure 9 are very small

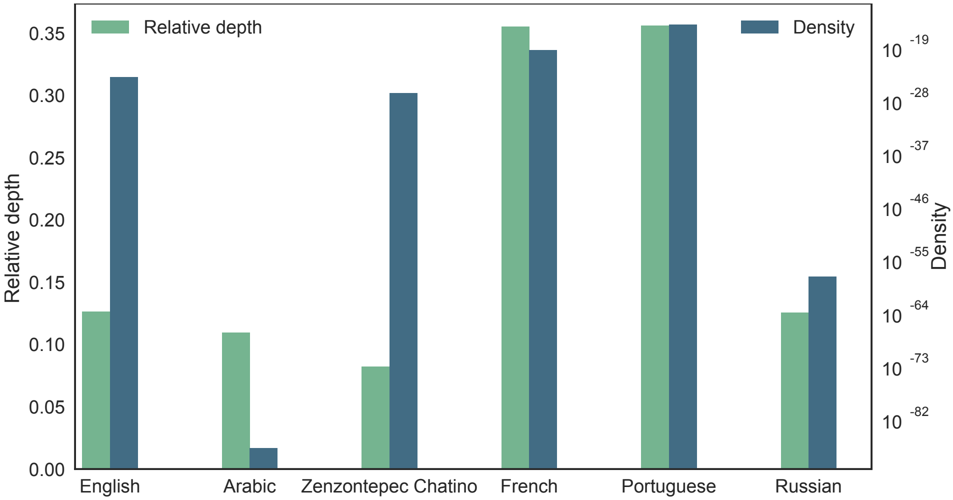


Figure 9: Relative canonicity measures on alternation pattern lattices

(they are shown on a log scale to make them perceptible), and depend mainly on the number of leaves. There is more variation in relative depth. In Zenzontepec Chatino, Arabic, Russian and English, relative depth is lower than 0.15, while Portuguese and French verbal systems have densities around 0.35, indicative of a more hierarchical system. It is interesting to note that the absolute depth in Russian, French and Portuguese is similar, but results in a higher density for Portuguese and French, because they have less than 100 microclasses, when Russian counts over 200. It appears that the French and Portuguese verbal systems, both romance languages, would be especially poorly accounted for by a partition, despite a tradition of doing so in romance linguistics.

Globally, these results show that the classifications we obtain are visually very complex, and far from the canon. This allows us to reject without hesitation the hypothesis according to which either partitions or tree structures would be appropriate models of inflection classes. However, these systems are also orders of magnitude less complex than the theoretical maximum.

Conclusion

In this chapter, we argued that while "Inflection classes" usually refers to either partitions or trees, the similarity structure of inflectional systems is usually more complex, and should rather be modeled as a lattice. Following the intuition according to which inflection classes are sets of lexemes distinguished by common inflectional properties, we put forward that any such maximal set is a relevant inflection class. Formal Concept Analysis allows us to build automatically the ordered set of all these classes, or *concepts*, from paradigms of alternation patterns inferred over large lexicon.

Using this methodology, we investigated the verbal systems of Modern Standard Arabic, English, French, European Portuguese, and Zenzontepec Chatino, and the nominal system of Russian. We find that in all cases, the similarity structure between inflectional paradigms is undoubtedly hierarchical, and that heteroclisis (multiple inheritance) is pervasive. These facts hold strongly even in systems like English which are usually seen as having a trivial inflectional structure.

The classifications we obtain are much larger than what is suggested by traditional accounts, and far too large for manual analysis. Usually, Inflection classes are taken to be convenient summaries of an inflectional system. Our investigation shows that this is not the case when taking into account the entire inflection class structure: the number of concepts is often of the same order, if not higher, as the size of the lexicon. While one can always choose a small subset of classes for pedagogical or constructive purposes, we see no prominent such subset in the hierarchies. This can certainly explain why there are so many alternative analyses of known inflectional systems into partitions of inflection classes.

We defined precise quantitative measures of inflectional canonicity, taking partitions and trees as two degrees of inflectional canonicity. We showed that while the systems are much larger than they would be in the canonical situation, they are much closer to that ideal than they are to the theoretical maximum. This indicates that these systems are certainly not arbitrarily complex. This finding goes along with known observations that inflectional complexity, while surprisingly high in appearance, is usually bounded (Carstairs 1987; Carstairs-McCarthy 1991; Ackerman, Blevins & Malouf 2009; Ackerman & Malouf 2015).

In conclusion, this study highlights the fact that the distribution of inflectional behaviors in a realistic lexicon is both highly structured and much more intricate than hand-crafted descriptions suggest.

Appendix

To compute inflection class lattices, we take as input paradigm tables of full, non segmented, raw forms in phonemic notation. The algorithm we use to infer alternation patterns (Beniamine 2018; 2017) also requires a decomposition of each phoneme in distinctive features. These are used to find more linguistically sound alternations using phonologically weighted similarities, and to choose patterns which lead to better generalizations over the whole lexicon. Unless specified otherwise, the definition of these features was based on Hayes (2012). The datasets and their constitution are described in more detail in (Beniamine 2018).

Modern standard Arabic is a central semitic language. It is the standardized variety of Arabic used in writing in Arabic speaking countries. Our lexicon was extracted and normalized from Wiktionnary entries as part of the UNIMORPH project (Kirov et al. 2016). The forms were transcribed phonemically semi-automatically (Beniamine 2018). Our lexicon counts 1018 lexemes, inflected for 109 possible combinations of mode, tense, voice, gender, person and number.

English is a West Germanic language spoken in the United Kingdom, the United States, Australia, Canada, and globally as a *lingua franca*. Our lexicon is a subset of the CELEX2 database (Baayen, Piepenbrock & Gulikers 1995). The original SAMPA notations were transcribed into IPA automatically (Beniamine 2018). The original lexicon often includes regional variants, which leads to paradigms where overabundance (more than one form for a given lexeme and paradigm cell) is frequent. Most verbs are inflected for five paradigm cells: present third person, other present forms, past participle, present participle, past. However, because of the verb TO BE, which is overdifferentiated, we count eight paradigm cells: infinitive, present first person, present third person, present other persons, past participle, present participle, past first person, past third person, other past persons. The lexicon counts 6064 verbal lexemes. Distinctive phonological features are based on Halle & Clements (1983) and Chomsky & Halle (1968).

French is a Romance language spoken primarily in France. French verbs are inflected for 51 paradigm cells, structured in seven finite tenses each inflected for six persons, the imperative inflected for only two persons, and six non finite cells. We use the verbal entries from the lexicon Flexique (Bonami, Caron & Plancq 2014), itself based on Lexique (New et al. 2001). Phonological features are based on Dell (1973). The lexicon counts 5249 lexemes.

European Portuguese is a Romance language spoken in Portugal. Our lexicon is based on frequent verbs from Veiga, Candeias & Perdigão's (2013) pronunciation dictionary. It counts 1996 lexemes inflected for 69 combinations of mood, tense and person. Phonological features originate from Bonami & Luís (2014).

Russian is an East Slavic language spoken in Russia and neighboring countries. Our lexicon was generated as romanized forms derived from the Network Morphology lexicon of Russian nouns described in Brown & Hippisley (2012), and transcribed phonemically semi automatically (Beniamine 2018). The nominal paradigm of Russian counts six combinations of case and number. Our lexicon counts 1529 lexemes. A small number of lexemes are also inflected for second singular locative (see Brown 2007).

Zenzontepec Chatino is a Chatino language of the Zapotecan branch of Oto-Manguean, spoken in Oaxaca, Mexico. The dataset we use comes from Surrey's Oto-Manguean Inflectional Class Database (Feist & Palancar 2015) and is based on data provided by Eric Campbell. Implicit low tones were added automatically in the dataset (Beniamine 2018). Zenzontepec Chatino verbs are inflected for only four paradigm cells, with aspect/mood values: completive, potential, habitual and progressive. The dataset counts 324 lexemes.

Acknowledgements

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